**Barron’s Let’s Review Regents – Algebra I**

# Chapter 4: Systems of Linear Equations

## 4.1 Solving Systems with Guess and Check

### Finding an Ordered Pair That Satisfies an Equation with Two Variables

### Checking to See If an Ordered Pair Is Part of the Solution Set of a Two-Variable Equation

### Identifying Equations That Have Equivalent Solution Sets

### Solving Systems of Equations by Guess and Check

The guess and check method is not very practical unless it is a multiple-choice question where there are at most four ordered pairs to check.

### Check Your Understanding of Section 4.1

1. Multiple-Choice
2. (2, 5) is a solution to which equation?  
   2(2) + 5 = 9 ck.  
   **(2) 2x + y = 9**
3. Which ordered pair is a solution to the equation 3y – 4x = 3?  
   (1) (3, 5) => x = 3, y = 5  
   3(5) – 4(3) ≟ 3  
   15 – 12 = 3 ck  
   **(1) (3, 5)**
4. Which ordered pair is ***not*** a solution to the equation 3y – 2x = 3?  
   **(4) (10, 8) => x = 10, y = 8**3(8) – 2(10) ≟ 3  
   24 – 20 3 (not a solution)
5. Which equation has the same solution as the equation 2x + 3y = 5?  
   Multiply by 4: 8x + 12y = 20  
   **(1) 8x + 12y = 20**
6. Which equation does not have the same solution as the equation 4x – 8y = 12?  
   **(1) 6x – 7y = 20**
7. Which is the solution set to this system of equations?  
   2x + 3y = 20  
   5x – 2y = 31  
     
   (1) (4,4)  
   2(4) + 3(4) ≟ 20  
   8 + 12 = 20 ck  
   5(4) – 2(4) ≟ 31  
   20 – 8 failed  
     
   (2) (1, 6)  
   2(1) + 3(6) ≟ 20  
   2 + 18 = 20 ck  
   5(1) – 2(6) ≟ 31  
   5 – 12 31 – failed  
     
   (3) (-2, 8)  
   2(-2) + 3(8) ≟ 20  
   -4 + 24 = 20 ck  
   5(-2) – 2(8) ≟ 31  
   -10 – 16 31 failed  
     
   **(4) (7,2)**2(7) + 3(2) ≟ 20  
   14 + 6 = 20 ck  
   5(7) – 2(2) ≟ 31  
   35 – 4 = 31 ck
8. Which is the solution set to this system of equations?  
   -4x + 2y = 18  
   3x – 6y = -36  
     
   **(1) (-2, 5)**-4(-2) + 2(5) ≟ 18  
   8 + 10 = 18 ck  
   3(-2) – 6(5) ≟ -36  
   -6 -30 = -36 ck
9. The ordered pair (3, -7) is a solution to which system of equations?  
   **(3) 2x – 3y = 27, 4x + 2y = -2**  
   2(3) -3(-7) ≟ 27  
   6 + 21 = 27 ck  
   4(3) + 2(-7) ≟ -2  
   12 – 14 = -2 ck
10. If (a, 5) is a solution to the equation   
    3x + 6y = 42, what is the value of a?  
      
    3(a) + 6(5) = 42  
    3a + 30 = 42  
    -30 = -30  
    3a = 12  
    a = 4  
      
    **(4) 4**
11. Which is not a solution to the system of equations?  
    x + y = 12  
    2x + 2y = 24  
    **(2) (3,10)**  
    3 + 10 12
12. Show how you arrived at your answers.
13. The equation 2x + 3y = 11 has two solutions in which both coordinates are positive integers less than 6. What are those two solutions?  
      
    (1, 3) and (4, 1)  
      
    x = 1  
    2(1) + 3y = 11  
    -2 = -2  
    3y = 9  
    y = 3  
      
    x = 4  
    2(4) + 3y = 11  
    8 + 3y = 11  
    -8 = -8  
    3y = 3  
    y = 1
14. Use guess and check to find the solution to the system of equations.  
    x + y = 8  
    x – y = 6  
    Ordered pairs that satisfy the first equation:  
    (1, 7), (2, 6),(3, 5), (4, 4), (5, 3), (6, 2), (7, 1)  
      
    **(7, 1)**7 + 1 = 8 ck  
    7 – 1 = 6 ck
15. The system of equations:  
    2x + 5y = 25  
    2x + 5y = 26  
      
    has no solutions. Explain why.  
      
    **The two equations are contradictory and any proposed solution to one equation would result in a failed check in the other equation.**
16. The equation x + 3y = c has the point (4,7) in its solution set. What is c?  
      
    4 + 3(7) = 4 + 21 = 25  
    **c = 25**
17. (6, -2) is a solution to the equation   
    5x – 6y = 42.   
      
    Find a solution to 15x – 18y = 126  
      
    5x – 6y = 42  
    Multiply by 3  
    15x -18y = 126  
      
    Therefore (6, -2) is a solution to the second equation as well as the first, since the second equation matches the first equation when multiplied by 3.

## 4.2 Solving Simpler Systems of Equations with Algebra

Certain systems of equations can be solved by combining the two equations such tat one of the variables gets eliminated. Three ways of doing this are addition, when the two equations are added to each other; subtraction, when one equation is subtracted from the other; and substitution, when one of the variables in one equation is replaced by an expression involving the other variable.

### Combining Two Equations to Form a New Equation

2x + 5y = 26  
3x – 2y = 1  
---------------  
5x + 3y = 27

This new equation is satisfied by the ordered pair (3,4).  
5(3) + 3(4) = 27  
15 + 12 = 27  
27 27

### Adding Two Equations to Eliminate a Variable

5x + 2y = 36  
3x – 2y = 12  
---------------  
8x = 48  
x = 6  
5(6) + 2y = 36  
30 + 2y = 36  
-30 = -30  
2y = 6  
y = 3  
  
x = 6, y = 3  
3(6) – 2(3) = 12  
18 – 6 = 12 ck

### Subtracting Two Equations to Eliminate a Variable

4x + 5y = 37  
-(2x + 5y = 31)  
---------------  
2x = 6  
x = 3  
4(3) + 5y = 37  
12 + 5y = 37  
-12 = -12  
5y = 25  
y = 5  
x = 3, y = 5

### Solving Systems of Equations with the Substitution Method

y = 2x + 1  
3x + 2y = 16  
3x + 2(2x + 1) = 16  
3x + 4x + 2 = 16  
7x + 2 = 16  
-2 = -2  
7x = 14  
x = 2  
y = 2(2) + 1  
y = 5  
x = 2, y = 5

### Check Your Understanding of Section 4.2

1. Multiple-Choice
2. Which equation has, as one of it solutions, the solution to the system of equations?  
   **(4) 7x – 3y = 24**
3. Solve the system of equations.  
   3x + 2y = 17  
   4x – 2y = 4  
   -------------  
   7x = 21, x = 3  
   3(3) + 2y = 17  
   9 + 2y = 17  
   -9 = -9  
   ----------  
   2y = 8, y = 4  
   x = 3, y = 4 => (3, 4)  
   **(3) (3,4)**
4. Solve the system of equations.  
   y = 5x + 3  
   2x + 6y = 50  
   2x + 6(5x + 3) = 50  
   2x + 30x + 18 = 50  
   -18 = -18  
   32x = 32  
   x = 1, y = 5(1) + 3 = 8  
   **(1) (1, 8)**
5. Solve the system of equations.  
   6x – 5y = 19  
   -3x + 5y = -7  
   ----------------  
   3x = 12, x = 4  
   6(4) – 5y = 19  
   5y - 19 = 5y - 19  
   ---------------------  
   24 – 19 = 5y, 5y = 5, y = 1  
   x = 4, y = 1   
   **(4) (4, 1)**
6. Solve the system of equations.  
   2x + 3y = -1  
   -2x + 5y = -23  
   ---------------  
   8y = -24, y = -3  
   2x + 3(-3) = -1  
   2x - 9 = -1  
   9 = 9  
   2x = -1 + 9 = 8, x = 4  
   x = 4, y = -3  
   **(3) (4, -3)**
7. Solve the system of equations.  
   -3x + 7y = 1  
   3x + 3y = -21  
   ----------------  
   10y = -20, y = -2  
   -3x + 7(-2) = 1  
   -3x – 14 = 1  
   14 = 14  
   ---------------  
   -3x = 15, x = -5  
   -3(-5) + 7y = 1  
   15 + 7y = 1  
   -15 = -15  
   --------------  
   7y = 1 – 15 = -14, y = -2  
   x = -5, y = -2  
   **(3) (-5, -2)**
8. Solve the system of equations.  
   3x + 7y = -2  
   x – 7y = -10  
   7y = 7y  
   x = 7y – 10  
   3(7y – 10) + 7y = -2  
   21y – 30 + 7y = -2  
   30 = 30  
   28y = 28, y = 1  
   3x + 7(1) = -2  
   -7 = -7  
   3x = -2 – 7 = -9, x = -3  
   x = -3, y = 1  
   **(4) (-3, 1)**
9. Solve the system of equations.  
   5x + 3y = 19  
   -(2x + 3y = 4)  
   -----------------  
   3x = 19 – 4 =15, x = 5  
   5(5) + 3y = 19  
   25 + 3y = 19  
   3y = 19 – 25 = -6, y = -2  
   x = 5, y = -2  
   **(1) (5, -2)**
10. Solve the system of equations.  
    y = 3x – 2  
    4x – 2y = -4  
      
    4x – 2(3x – 2) = -4  
    4x – 6x + 4 = -4  
    -4 = -4  
    -2x = -8, x = 4  
    y = 3x – 2 = 3(4) – 2 = 12 – 2 = 10  
    x = 4, y = 10  
    **(2) (4, 10)**
11. Solve the system of equations.  
    4x + 8y = 36  
    -(4x + 5y = 33)  
    ------------------  
    3y = 3, y = 1  
    4x + 8(1) = 36  
    -8 = -8  
    --------------------  
    4x = 36 – 8 = 28, x = 7  
    4(7) + 8y = 36  
    28 + 8y = 36, 8y = 8, y = 1  
    x = 7, y = 1  
    **(1) (7, 1)**

B. Show how you arrived at your answers.

1. Find an equation that has, as one of its solutions, the solution to the system of equations.  
   3x + 4y = 11  
   2x + 6y = 14  
   ---------------  
   **5x + 10y = 25**x = 1, y = 2  
   (1, 2)
2. A student likes to use the substitution method for systems of equations. How can he use it with a system that is not in the proper form for substitution/ Show with this system.  
   -2x + y = 4  
   3x + 4y = 49  
     
   -2x + y = 4  
   2x = 2x  
   y = 2x + 4  
     
   3x + 4(2x + 4) = 49  
   3x + 8x + 16 = 49  
   -16 = -16  
   11x = 33, x = 3  
   y = 2(3) + 4 = 6 + 4 = 10  
   x = 3, y = 10  
   **Solution: (3, 10)**
3. The system  
   y = 2x – 9  
   y = -3x + 16  
   can be solved in many ways.  
     
   Student 1 wants to use subtraction. Student 2 wants to use substitution. Show how each student would do this.  
     
   **Student 1, subtraction**  
   y = 2x – 9  
   -(y = -3x + 16)  
   -----------------  
   0 = 5x – 25, 5x = 25, x = 5  
   y = 2(5) – 9 = 10 – 9 = 1  
   **x = 5, y = 1**  
     
   **Student 2, substitution**  
   y = 2x – 9  
   y = -3x + 16  
     
   2x – 9 = -3x + 16  
   3x – 16 = 3x – 16  
   --------------------  
   5x – 25 = 0, 5x = 25, x = 5  
   y = 2(5) – 9 = 10 -9 = 1  
   **x = 5, y = 1**
4. Two numbers, x and y, have a sum of 18 but a difference of 11. Write a system of equations that can be used to solve this and then use it to find the solution.  
     
   x + y = 17  
   x – y = 11  
   -----------  
   2x = 17 + 11 = 28, x = 14  
   14 + y = 17  
   -14 = -14  
   y = 3  
   **x = 14, y = 3**
5. The system of equations  
   5x + 2y = 8  
   -5x + 2y = -32  
   can be solved several ways.  
     
   Student 1 wants to use addition. Student 2 wants to use subtraction. Who is right? Explain your answer.  
     
   Both strategies work well. Addition will eliminate the x value and allow solving for y. Subtraction will eliminate the y value allow solving for x.  
   **Student 1, addition**  
   4y = 8 – 32 = -24, y = -6  
   5x + 2(-6) = 8  
   5x – 12 = 8, 5x = 8 + 12 = 20, x = 4  
   x = 4, y = -6  
   **Student 2, subtraction**  
   10x = 8 – (-32) = 40, 10x = 40, x = 4  
   -5(4) + 2y = -32  
   -20 + 2y = -32  
   20 = 20  
   2y = -12, y = 06  
   **x = 4, y = -6**

## 4.3 Solving More Complicated Systems of Equations with Algebra

### Solving a System by Changing One of the Equations

3x + 4y = 31  
5x – 2y = -9

Adding or subtracting the two equations will not eliminate one of the variables.

You are permitted to multiply either equation by whatever number you want.

### Solving a System by Changing Both Equations

Sometimes the only way to change the equations so that a variable will drop out when adding them is to change both equations. This is needed when none of the coefficients is evenly divisible by the corresponding coefficient in the other equation.

### Systems of Equations That Have No Solutions or Infinite Number of Solutions

For some systems of equations, there are no ordered pairs that satisfy both equations. An example of this is:

x + y = 10  
x + y = 11

Any ordered pair that satisfies the first equation will not satisfy the second equation, and any ordered pair that satisfies the second equation will not satisfy the second equation.

The system of equations:

x + y = 10  
2x + 2y = 20

also has something unusual occur.

-2(x + y) = -2(10)  
-2x – 2y = -20  
2x + 2y = 20  
0 = 0

Since 0 does equal 0, the original system has an infinite number of solutions.

### Check Your Understanding of Section 4.3

1. Multiple Choice
2. Solve the system of equations:  
   2x + 3y = 16  
   5y = 10, y = 2  
     
   2x + 3(2) = 16  
   2x + 6 = 16  
   -6 = -6  
   2x = 10, x = 5  
   x = 5, y = 2  
   **(4) 5, 2)**
3. In order to eliminate the **y** from this system of equations,  
     
   3x + 12y = 21  
   6x + 4y = 22  
     
   what could you do?  
   **(2) Multiply both sides of the second equation by -3.**
4. In order to eliminate the x from this system of equations:  
   12x – 3y = 21  
   -2x + 6y = 2  
   what could you do?  
   **(2) Multiply both sides of the second equation by 6.**
5. Which step would not cause a variable to be eliminated after adding equations together in this system?  
     
   3x – 6 = 6  
   9x + 2y = 38  
     
   **(4) Multiply both sides of the first equation by 4.**
6. What is one way to eliminate the y from this system of equations?  
   3x + 4y = 16  
   2x – 5y = 3  
   (3) Multiply both sides of the first equation by +5 and both sides of the second equation by +4.
7. Solve the system of equations:  
   8x – 2y = 28  
   4x + 3y = 6  
     
   -2(4x + 3y) = -2(6)  
   -8x – 6y = -12  
   8x – 2y = 28  
   -8y = 16  
   y = -2  
   4x + 3(-2) = 6  
   4x – 6 = 6  
   6 = 6  
   4x = 6 + 6 = 12  
   x = 3  
   x = 3, y = -2  
   **(4) (3, -2)**
8. Solve the system of equations:  
   4x – 3y = 31  
   3x + 5y = 16  
     
   3(4x – 3y) = 3(31)  
   12x – 9y = 93  
     
   -4(3x + 5y) = -4(16)  
   -12x – 20y = -64  
   12x – 9y = 93  
   -29y = 29  
   y = -1  
   4x – 3(-1) = 31  
   4x + 3 = 31  
   -3 = -3  
   4x = 31-3 = 28  
   x = 7  
   x = 7, y = -1  
   **(1) (7, -1)**
9. Solve the system of equations:  
   x – 3y = 14  
   4x + 5y = 22  
     
   -4(x – 3y) = -4(14)  
   -4x + 12y = -56  
   4x + 5y = 22  
   17y = -34  
   y = -2  
   x – 3(-2) = 14  
   x + 6 = 14  
   x = 8  
   x = 8, y = -2  
   **(4) (8, -2)**
10. Solve the system of equations:  
    3x – 2y = -4  
    5x – 4y = -12  
      
    -2(3x – 2y) = -2(-4)  
    -6x + 4y = 8  
    5x – 4y = -12  
    -x = -4  
    x = 4  
    3(4) – 2y = -4  
    2y + 4 = 2y + 4  
    12 + 4 = 2y  
    2y = 16, y = 8  
    x = 4, y = 8  
    **(1) (4, 8)**
11. What is true about this system of equations?  
    2x – 7y = 2  
    6x – 21y = 7  
      
    3(2x – 7y) = 3(2)  
    6x – 21y = 6  
    6x – 21y = 7 (contradiction)  
    **(4) No ordered pairs solve the system.**
12. Show how you arrived at your answers.
13. For the system:  
    2x + 5y = 17  
    6x + 7y = 27  
    Samuel want to eliminate the y, but Georgia wants to eliminate the x. Which student’s choice will require less work?  
      
    **Eliminating x will require less work, as only the first equation will need to be multiplied by -3. Eliminating y will require more work, with the first equation having to be multiplied by 7, and the second equation by -5.**
14. Solve for x and y in this system:  
    x + y = 40  
    2x + 4y = 136  
      
    -2(x + y) = -2(40)  
    -2x – 2y = -80  
    2x + 4y = 136  
    2y = 56  
    y = 28  
    x + 28 = 40, x = 12  
    **x = 12, y = 28**
15. Aubree solve the system of equations:  
    2x + 3y = 21  
    4x + 6y = 42  
    and get the solution:  
    0 = 0  
    She says this means that every ordered pair is a solution to the system of equations. Is this accurate?  
    Multiplying the first equation by 2 produces the same exact equation as the second equation.  
    2(2x + 3y) = 2(21)  
    4x + 6y = 42  
      
    **It would be more accurate to say there is an infinite number of solutions than any arbitrary ordered pair is a solution to the system of equations.  
      
    The ordered pair (0,0) does not satisfy either equation. But every ordered pair that satisfies the first equation will satisfy the second equation.**
16. If it is possible, find an ordered pair that is a solution to the system.  
    5x – 2y = 5  
    10x -4y = 11  
    Multiplying the first equation by 2:  
    2(5x – 2y) = 2(5)  
    10x – 4y = 10  
    This contradicts the second equation:  
    10x – 4y = 11.  
    **No solution that satisfies the first equation will satisfy the second. There are no solutions.**  
    For example, x =1, y = 0, satisfies the first equation:  
    10(1) – 4(0) = 10  
    10 – 0 = 0  
    10 = 10  
    But x = 1, y = 0 does not satisfy the second equation:  
    10(1) – 4(0) 11  
    10 – 0 11  
    10 11
17. For the system of equations:  
    4x – 3y = 18  
    2x + 12y = 36  
    Victoria starts by multiplying both sides of the second equation by -2. Porter starts by multiplying both sides of the second equation by +2. How is it that either of these methods will lead to the correct solution?  
      
    Victoria can use addition to eliminate x and solve for y. Porter can use subtraction to eliminate x and solve for y. Both approaches will lead to a solution.